Pergamon

Int. J. Multiphase Flow Vol. 23, No. 4, pp. 671–682, 1997

Printed in Great Britain
0301-9322/97 $17.00 + 0.00

PII: S0301-9322(97)00009-8

INTERMITTENT TRANSITION FROM BUBBLING TO JETTING REGIME IN GAS–LIQUID TWO PHASE FLOWS

M. C. RUZICKA[1], J. DRAHOŠ[1], J. ZAHRADNÍK[1] and N. H. THOMAS[2]

[1]Institute of Chemical Process Fundamentals, Academy of Sciences of the Czech Republic, Rozvojova 135, 16502 Prague, Czech Republic

[2]FAST Team, School of Chemical Engineering and FRED Ltd, Research Park, University of Birmingham, Birmingham B15 2TT, U.K.

(Received 18 November 1996; in revised form 5 March 1997)

Abstract—The transition from bubbling to jetting regime in nitrogen–water system was studied experimentally. The gas was introduced into a pool of stagnant liquid through a single orifice plate above a gas chamber. Two quantities were measured: pressure fluctuations in the gas chamber and velocity of liquid circulations near the orifice. Individual bubbles were formed at low gas flow rates (bubbling regime) while a continuous jet of gas was formed at high rates (jetting regime). The transition from bubbling to jetting regime (transition regime) displayed intermittent character. Jetting bursts of various length appeared at random in originally periodic pressure signal. The distribution of bubbling portion in the pressure signal was hyperbolical with exponent −1.33 indicating type III intermittency. Similar characteristic time scales were found in power spectra of both signals. $1/f$ noise was revealed in the velocity spectrum. This kind of noise usually accompanies intermittent transitions. These results implied that liquid circulations with $1/f$ noise induced by bubbles affected the bubble dynamics itself as a feed-back and caused the intermittent regime transition. The point of the regime transition was indicated by a sudden drop of Kolmogorov entropy, correlation dimension of the attractor, and Mann–Whitney statistic calculated from pressure signal. An explanation for this drop is suggested on the base of combination of properties of two attractors coexisting/competing within the intermittency range. © 1997 Elsevier Science Ltd.

Key Words: bubble formation, pressure fluctuations, liquid circulations, hydrodynamic interaction, regime transition, intermittency, $1/f$ noise, Kolmogorov entropy, correlation dimension of attractor

1. INTRODUCTION

The hydrodynamics of bubble columns is very complex. Despite long time efforts (e.g. Deckwer 1992; Kastanek *et al.* 1993), many problems have not been understood yet. Hydrodynamic interactions between the gas and liquid phases are of crucial importance. It is not clear how the dynamic complexity of the two-phase flow is built up from local interactions among bubbles, what is the character of induced liquid circulations and how they affect the formation of bubbles at the gas distributor, what is the origin of hydrodynamic regimes transitions, and where the stochasticity and behavioural unpredictability is coming from.

These questions are difficult to answer even in the simple case of bubbling from a single orifice. The process of bubble formation was reviewed a decade ago by Tsuge (1986). Recently, the concept of nonlinear dynamic systems has been applied to study chaotic features of complex bubble formation dynamics (Tritton and Egdell 1993; Mittoni *et al.* 1995; Drahoš *et al.* 1996), but only at rather descriptive level. There are also studies on the effect of liquid motion on the bubble formation (Marshall *et al.* 1993; Oguz and Prosperetti 1993; Kim *et al.* 1994), but usually only a simple case of the effect of uniform and unidirectional stream of liquid on resulting bubble volume has been considered. The duality between bubbling and jetting regimes has been pointed out (Rabiger and Vogelpohl 1983, 1986), but the regimes transition itself has not been studied. Structural development of air–water plumes has been examined (Castillejos and Brimacombe 1987; Turkoglu and Farouk 1996), but there is a lack of information on far-field liquid motion induced by a buoyant gas jet in a bounded region.

This paper presents an experimental study on the transition from bubbling to jetting regime in the case of a single orifice plate above a gas chamber in the presence of natural self-induced liquid circulations. The time trace of the gas chamber pressure characterizes the bubble formation process (figure 1). The bubble expansion is indicated by the pressure drop. Bubble usually detaches when the pressure reaches the lowest value. The typical scenario of bubble formation is as follows. At low gas flows, individual bubbles (single, double, or clusters) are released from the orifice—*the bubbling regime*. The corresponding pressure signal is periodic but not necessarily harmonic. With increasing gas flow, the bubbling regime loses its stability. The signal is slightly modulated by a low frequency (figure 1(a)), nonuniformities appear both in the amplitude and period (figure 1(b)), and the signal is occasionally interrupted by low amplitude intermittent bursts (figure 1(c)). These bursts correspond to a continuous jet of gas at the orifice—*the jetting regime*. The number of jetting bursts and their length (their duration, in s) increases with increasing gas flow (figure 1(d)). The gas jet is eventually stabilized and the signal is irregular and aperiodic (figure 1(e)). There is a range

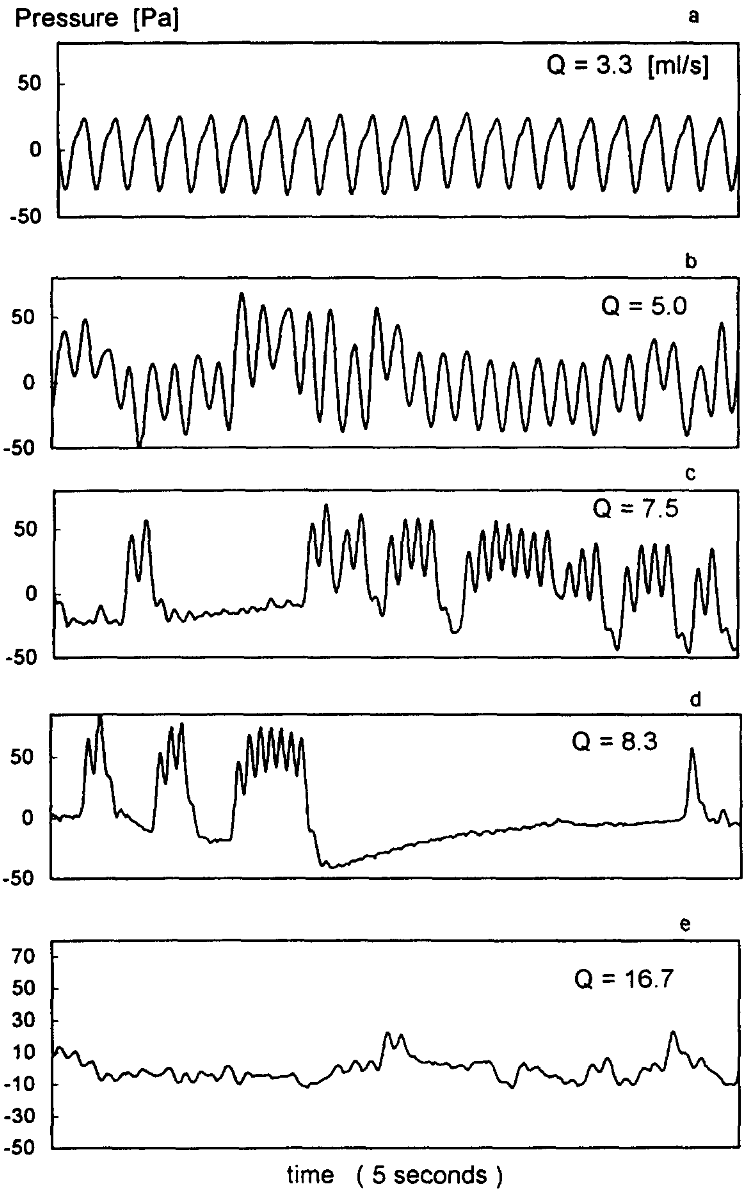


Figure 1. Character of gas chamber pressure signal when increasing gas flow rate Q: (a), (b) bubbling regime; (c), (d) intermittent transition regime; (e) jetting regime. ($H = 0.5$ m.)

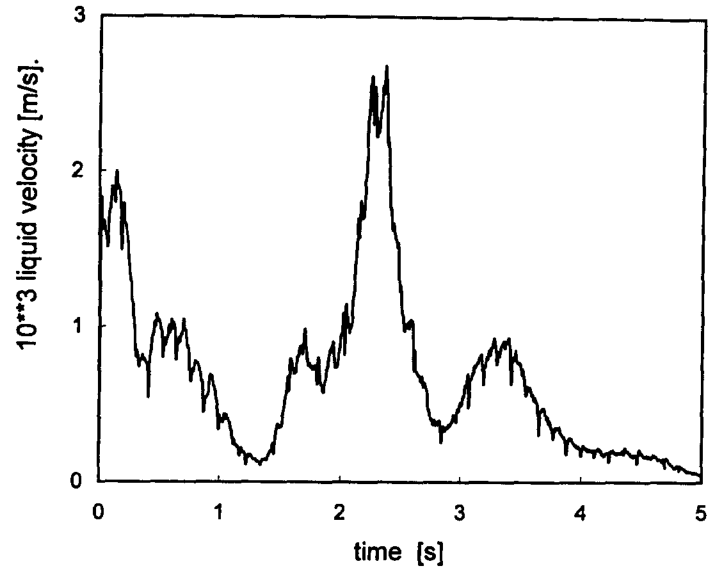


Figure 2. Typical liquid circulation velocity signal of in the column near the plate. ($R = 0.03$ m, $H = 0.5$ m, $Q = 6$ ml/s.)

of gas flow rates where both regimes coexist and compete for the input energy. The following three factors contribute to the destabilization of the bubbling regime:

(1) gas chamber–bubble interactions
(2) bubble–bubble interactions
(3) liquid flow–bubble interactions.

The presence of the gas chamber between the pressure source and the orifice makes the bubble formation dynamics very complex (case 1). Local hydrodynamic interactions between successive bubbles due to bubble wakes become important at high gas flow rates where the bubble spacing is low (case 2). The third interaction is caused by the overall liquid motion induced by the buoyant rise of bubbles. The upflow of liquid must be compensated by the downflow and large scale motion (macrocirculation) is set up in a bounded region, the column. The circulations are dynamic and nonstationary in nature. The occurrence of a liquid stream at a given place and in a given time is random. A typical velocity time trace is shown in figure 2. It is assumed that the circulations interfere with the bubble formation process. Ideally, a strong enough liquid stream passes over the orifice and breaks the periodic chain of bubbles. The chamber pressure rises and then the gas is released in the form of a jet.

The goal of this study is to explore the character of the regime transition and the interaction between gas and liquid phase. The circulation velocity was monitored by a hot film anemometer probe near the orifice and the chamber pressure was monitored by a pressure transducer. The behaviour of circulations was studied and the effect of three parameters was considered (radial probe position, water depth, and gas flow rate). Both velocity and pressure signals were analysed to elucidate the process of regime transition and phase interaction.

2. EXPERIMENTAL

2.1. Apparatus

Experiments were performed with nitrogen and tap water (typically at 293 K) in a plexiglas column (0.14 m i.d., 1 m high) equipped with a plate above the gas chamber (0.0004 m^3). The brass plate was 0.003 m thick with only one circular orifice (0.0016 m) at the centre. Chamber pressure was measured with a transducer ZPA11447 and the liquid velocity with a Dantec anemometer system (CTA 56C01, bridge 56C17). The sensor of the probe (90°-bent unidirectional wedge-shaped probe 55R34) was placed 0.02 m above the plate and directed upwards to indicate preferably downcoming liquid streams. The probe was moveable in the radial direction.

2.2. *Data acquisition and treatment*

Voltage outputs from the velocity and pressure probes passed through a low pass analogue filter (typically 40–50 Hz) to the 16 bit A/D converter (sampling frequency 100–400 Hz). Digitized time series $v(t)$ and $p(t)$ were stored in the PC. To reveal the underlying structure of the data, the following quantities were calculated.

- Mean and variance of the velocity signal, to characterize the intensity of the liquid motion.
- Intermittency factor γ of jetting bursts in the pressure signal, to locate the regime transition. γ equals to the relative length of jetting bursts in pressure signal, i.e. (total time of jetting portions in the signal)/(signal length, in seconds), precision 1–2%. A simple programme was used to distinguish between the bubbling and jetting portions of the signal. It was based on the difference between amplitude values of both portions.
- Distribution $N(L)$ of the length L of jetting bursts, to describe the intermittent character of the transition (5 min of pressure signal were used for calculation of γ and $N(L)$; precision 1–2%).
- Kolmogorov entropy K (Schouten *et al.* 1994a), correlation dimension of the attractor D (Schouten *et al.* 1994b), and the Mann–Whitney statistic Z (Kennel and Isabelle 1992) of both signals were calculated. These quantities are used to describe properties of signals generated by chaotic dynamical systems. K and D are measures of complexity of the dynamic behaviour. Z-statistic measures the degree of randomness of stochasticity of the signal. The signal is likely generated by a random process if Z is higher than about -3 and by a deterministic process if Z is less than about -3. (RRCHAOS software, Schouten and van den Bleek 1996; calculations on 60 000 data points, i.e. 150 s; relative standard error less than 1%.)
- Power spectra $v^2(f)$ and $p^2(f)$ of both signals, to reveal characteristic time scales for liquid motion and bubble dynamics (Hanning data window was applied to suppress the leakage effect; the result is the average over 10 blocks, each of 8192 data points, i.e. 10×20 s). The overall scheme of measurements is given in table 1.

3. RESULTS AND DISCUSSION

3.1. *Liquid circulation*

Both the velocity mean and the variance displayed similar trends, see figures 3 and 4. The order of magnitude $\sim 10^{-3}$ m/s for the mean downward liquid velocity indicated by the probe was two orders less than maximum velocity of $\sim 10^{-1}$ m/s induced by bubble rise in the sparged central zone of the column (videorecord). The following factors could contribute to this difference: the ratio of (sparged/total) cross section due to the continuity equation, the three-dimensional nature of the velocity field compared with unidirectional velocity measurement, and viscous dissipation. Their relative importance is not discussed here. The radial velocity profile was rather flat (not shown).

Table 1. Scheme of measurements

Liquid circulations: effect of R, H, and Q				
Run	Measured	Calculated	Parameters	Results
1	$v(t)$, 200 Hz	Mean, var.	$R = 1$–5, $H = 0.5$, $Q = 2.5$–25	—
2	$v(t)$, 200 Hz	Mean, var.	$R = 3$, $H = 0.05$–0.90, $Q = 7$	Figure 3
3	$v(t)$, 200 Hz	Mean, var.	$R = 3$, $H = 0.5$, $Q = 2.5$–40	Figure 4
Liquid circulations and bubble dynamics: effect of Q ($R = 3$, $H = 0.5$)				
Run	Measured	Calculated	Parameter	Results
4	$p(t)$, 100 Hz	γ, $N(L)$	$Q = 6.3$–8.8	Figures 5, 6
5	$v(t)$, 400 Hz	K_v, D_v, Z_v	$Q = 2.5$–33	—
6	$p(t)$, 400 Hz	K_p, D_p, Z_p	$Q = 3$–12	Figure 7
7	$v(t)$, $p(t)$, 400 Hz	$v^2(f)$, $p^2(f)$	$Q = 5$–12	Figure 9

v—velocity, p—pressure, R—radial probe position [10^{-2} m], H—water depth in the column [m], Q—gas flow rate [ml/s], γ—intermittency factor, $N(L)$—jetting bursts length distribution, K—Kolmogorov entropy, D—attractor dimension, Z—Mann–Whitney statistic.

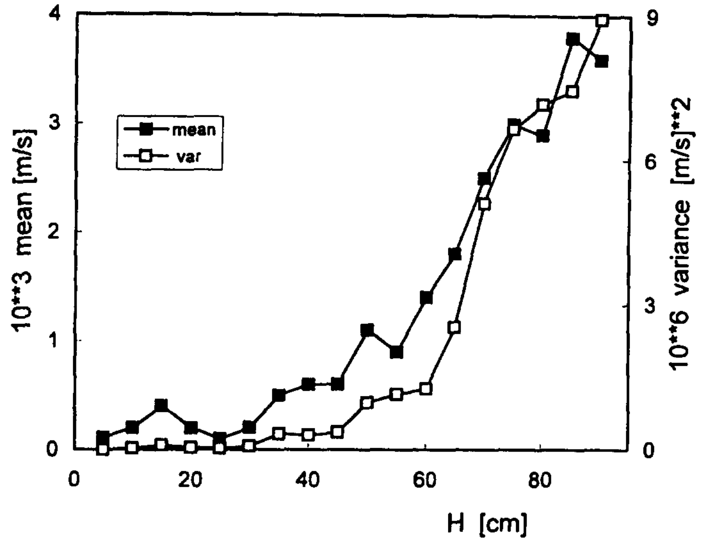


Figure 3. Dependence of liquid circulation velocity (mean and variance) on water depth H in the column. ($R = 0.03$ m, $Q = 7$ ml/s.)

The liquid motion displayed similar statistical values along the radial direction on the scale of few minutes and was spatially homogeneous. The depth of liquid column supported circulation (figure 3) and the flow followed the boundary represented by the wall. The character of the large scale motion can be changed by a suitable choice of the apparatus geometry (not only the aspect ratio, but also absolute length scales must be taken into account). A speculation can be made that relatively stable circulations (resembling convective patterns in thermal convection) could develop at some particular values of the aspect ratio. The axial velocity profile would be needed to draw a conclusion. The gas flow rate supported downward circulations only to a certain degree (figure 4). The input energy helped to build them at low gas flows and destroyed them at higher gas flows. This feature is common in physics of open systems. The existence of a given pattern of behaviour is possible only in certain parameter range (Haken 1977).

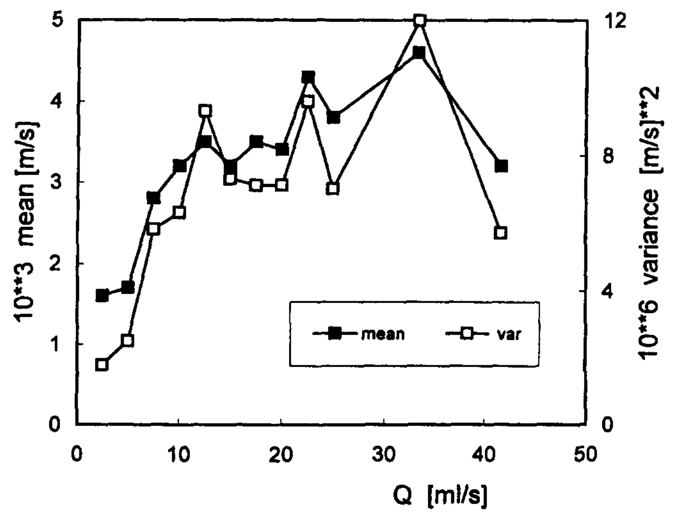


Figure 4. Dependence of liquid circulation velocity (mean and variance) on gas flow rate Q. ($R = 0.03$ m, $H = 0.5$ m.)

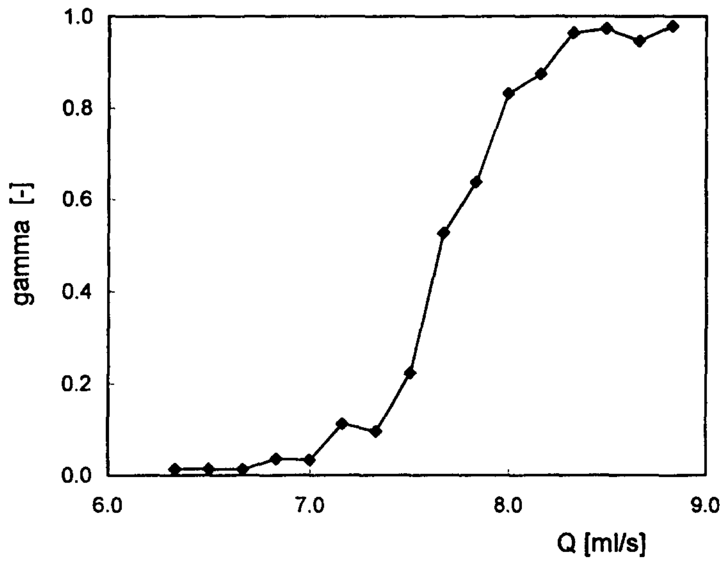


Figure 5. Intermittency factor γ indicates the transition from bubbling to jetting regime when increasing gas flow rate Q. ($R = 0.03$ m, $H = 0.5$ m.)

3.2. Character of regime transition

The bubbling-to-jetting regime transition was primarily indicated by the intermittency factor, figure 5. The transition was confined to a narrow range of the control parameter of about $Q = 7.0$–8.3 ml/s. Despite that, the jet could be occasionally interrupted even at much higher rates. Basically, there are three types of intermittency transitions from periodic to aperiodic behaviour. They correspond to three different kinds of bifurcations of the attractor found in simple dynamical systems, one-dimensional maps (Hilborn 1994; Schuster 1995). The different types can be, among others, distinguished by the shape of the distribution $N(L)$ of periodic (bubbling) portion length L in the intermittent signal (Schuster 1995): U-shaped distribution is typical for type I

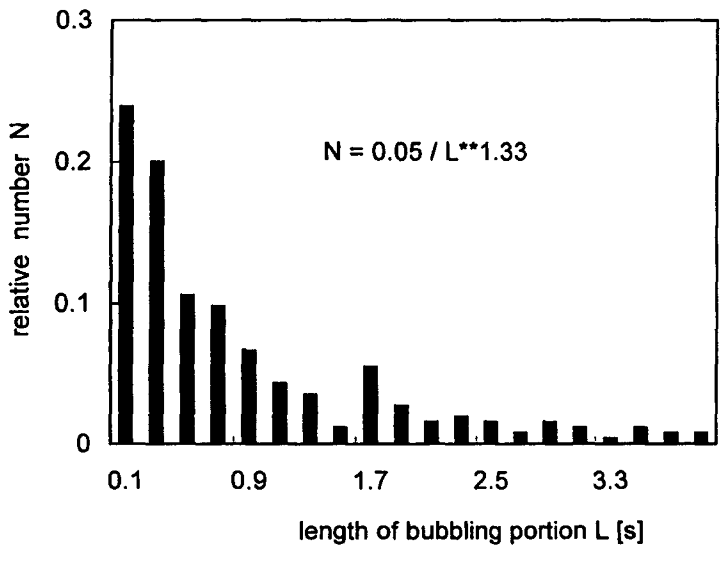


Figure 6. Distribution of bubbling portion in gas chamber pressure signal in the transition regime range. (Average over three distributions corresponding to three values of the parameter $Q = 7.50$, 7.67, and 7.83 ml/s; $R = 0.03$ *m*, $H = 0.5$ m.)

intermittency, while types II and III display hyperbolic distribution with exponents -2 and -1.5. Our data, figure 6, obey the hyperbolic law

$$N(L) \sim L^{-1.33} \qquad [1]$$

with the exponent -1.33 ± 0.13 and correlation coefficient of linearized log–log plot $r = -0.92$. Accordingly, this case is closer to type III that to type II intermittency. The corresponding distribution of aperiodic (jetting) portion length in pressure signal was also hyperbolic with exponent -1.82 ± 0.12.

Another characteristic feature of intermittency is the relation between the average length $\langle L \rangle$ of periodic portion in the signal and the bifurcation parameter Q (Schuster 1995). All three types follow hyperbolic law

$$\langle L \rangle \sim (Q - Q_c)^a, \qquad [2]$$

where Q_c is the staring point of the transition. Value of the exponent a is -0.5 for type I and -1 for types II and III. Value -2.10 ± 0.15 was found for our data (correlation coefficient -0.96), which is closer to types II and III. Note, that intermittency was classified on the ground of theoretical study of behaviour of very simple systems. Although the bifurcation phenomena bear fractures of universality, quantitative agreement between the theoretical prediction and real experimental data from far more complex systems is expected to be rare. Nevertheless, all three types of intermittency have been observed experimentally in the 'pure' form (Ringuet *et al.* 1993; Hilborn 1994; Schuster 1995). More detailed information about intermittency has been given, e.g. by Frisch (1995).

Chaotic dynamic quantities computed from pressure signals have been qualified as sensitive indicators of hydrodynamic regime transitions. They displayed a sudden and so far unexplained drop in the vicinity of the transition point, figure 7. This drop in Kolmogorov entropy was found, for instance, in the case of a simple particle array model of a fluidized bed (van den Bleek and Schouten 1993a) and for regime transitions both in fluidized beds (van den Bleek and Schouten 1993b) and gas–liquid systems (Drahoš *et al.* 1996; Letzel *et al.* 1996), where the pressure fluctuations were measured. This phenomenon bears some features of universality because it is independent of the nature of the particular system (neither the physical nature nor the complexity). An explanation for the drop occurrence is given here. Generally, one would not expect a drop of dynamic complexity in the case of hydrodynamic systems when increasing the input energy. The systems are driven from laminar to turbulent motion through a sequence of excitation of wilder and wilder dynamic modes. The occurrence of the drop means that the system tends to organize somehow near the transition point. At that point, in our case, control is transferred from the old bubbling attractor B (that has already reached the limit of its capability to reflect the dynamics) to the newly born jetting one J (that is more adequate but possibly still less complex than B). In the case of intermittency, the system parameters undergo slight variation around the exact bifurcation value due to not yet fully understood reasons, so called $1/f$ noise (Miller *et al.* 1993). The consequence is, that both B and J coexist and alternate in controlling the dynamic behaviour. Assuming that the resulting property of the intermittent signal can be expressed as a simple linear combination of properties of the two participating attractors, B and J, one can arrive at the relation

$$K_m = (1 - \gamma)K_B + \gamma K_J \qquad [3]$$

for the entropy of the intermittent mixture (B + J). To determine numerical values of K_B and K_J, two model series were extracted from intermittent pressure signals by eliminating almost all either jetting or bubbling portions (the model series was similar to signals at the beginning and the end of the transition). The following values were found:

$$K_B = 11.7 \quad \text{and} \quad K_J = 0.6 \quad \text{[bit/s]}.$$

The comparison of K_m calculated from [3] with the real signal entropy K is shown in figure 8. The fit of the descending branch of the graph is acceptable. Physically it means that the resulting dynamics of two competing modes can be approximated by a linear combination of dynamic characteristics of both modes in this case. Further increase of the entropy beyond the transition

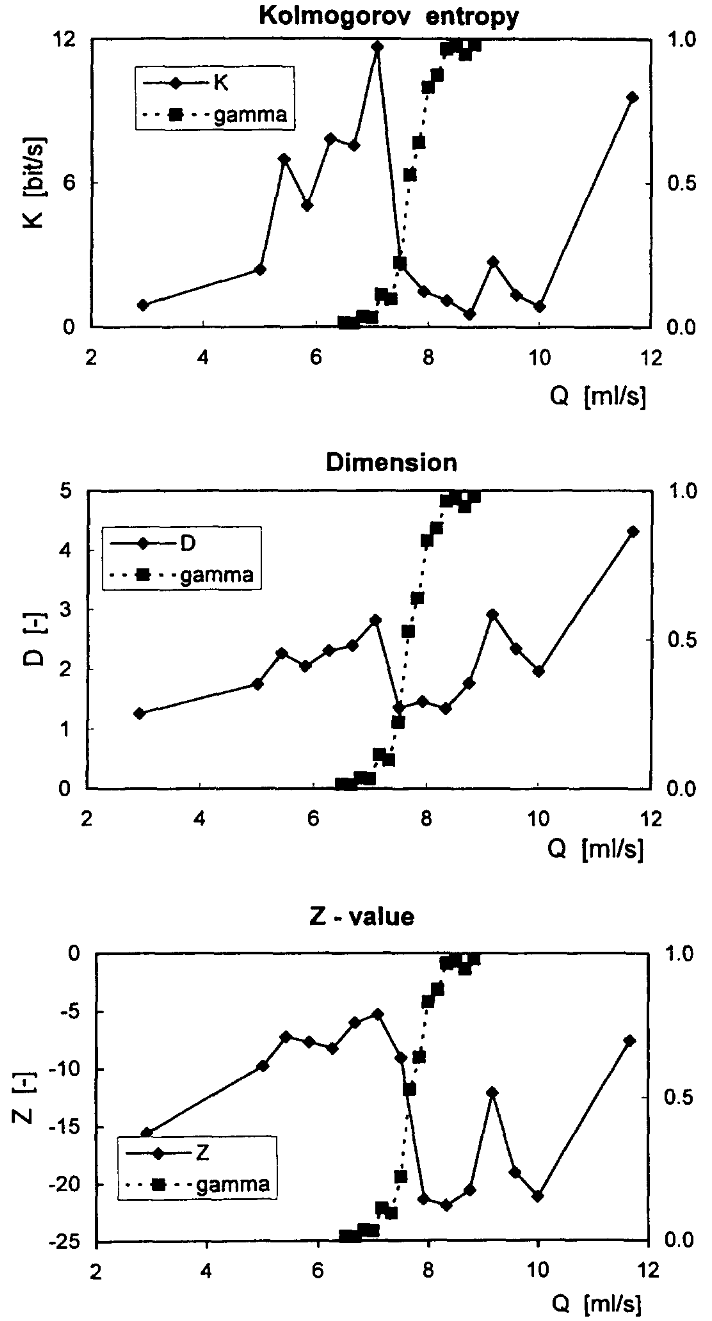


Figure 7. Dependence of Kolmogorov entropy K, correlation dimension D of the attractor, and Mann–Whitney statistic Z on gas flow rate Q. Intermittency factor γ indicates the transition region. ($R = 0.03$ m, $H = 0.5$ m.)

is due to natural development of the structure of the new attractor as the control parameter Q increases.

Besides bubbly flows and fluidized beds, the effect of the decrease in behavioural complexity has also been found in other hydrodynamic systems, for instance in Couette–Taylor flow (Brandstater and Swinney 1987) and thermal convection (Mukutmoni and Yang 1995). This phenomenon was observed also in the behaviour of simple quadratic maps (Baker and Gollub 1990; Hilborn 1994) where periodic windows appear within a chaotic region. But in the last case, the control parameter does not necessarily represent the input energy into the system.

Chaotic quantities computed from velocity signals had no noticeable trend. They displayed rather slight variation around a constant level within the parameter range tested (table 1, run 6, graphs not shown). Typical values were:

$$K \sim 2\ \text{bit/s}, \quad D \sim 3, \quad Z \sim -12.$$

The reason for this is likely that the Reynolds number (based on the linear gas velocity in the column and the column diameter) was rather small, Re = 22–290, too small to induce drastic changes in complexity of the liquid motion.

3.3. *Liquid circulations and bubble dynamics*

The pressures and velocity signals were compared to examine the effect of the liquid motion on the bubble dynamics. They could not be compared directly because circulations were indicated only in one direction and at one point, which was spatially distant from the orifice where the bubbling-jetting instability took place. The following findings provide indirect proof of gas–liquid phase interaction:

(1) Both intensity of liquid circulations and intermittency factor (figures 4 and 5) displayed the steepest increase in the transient region, $Q \sim 7$–8 ml/s.

(2) Common characteristic time scales were found in power spectra of the both signals, figure 9. At low gas flows, there were only peaks in the bubble frequency (up to about 10 Hz). Increasing the gas flow, the bubbling regime started to lose its stability and the signal displayed modulation, nonuniformities, and interruptions (figure 1). It was reflected in the spectrum as low frequency components (figure 9). At the same time, the velocity spectrum contained only low frequency components in the similar domain. This indicates that the liquid circulation and bubbling instability events operated on statistically similar time scales.

(3) $1/f^{\alpha}$ noise was discovered in liquid velocity spectra (figure 9) with the exponent $\alpha = 1.56$. The gas and liquid phases interacted in the column as follows. The gas phase induced buoyancy-driven large-scale liquid motion of random and low frequency character ($1/f$ noise). The liquid motion fed this $1/f$ noise into the gas phase dynamics and supported the intermittency in bubbling-jetting regime transition.

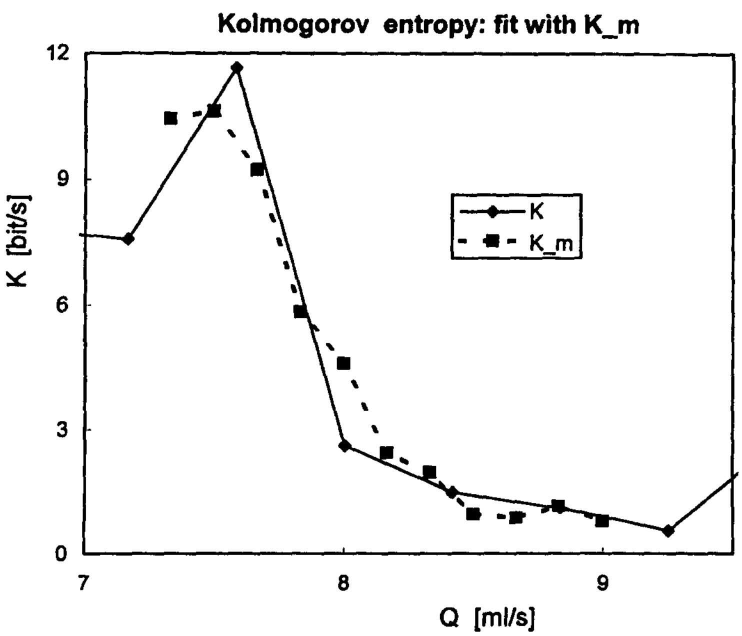
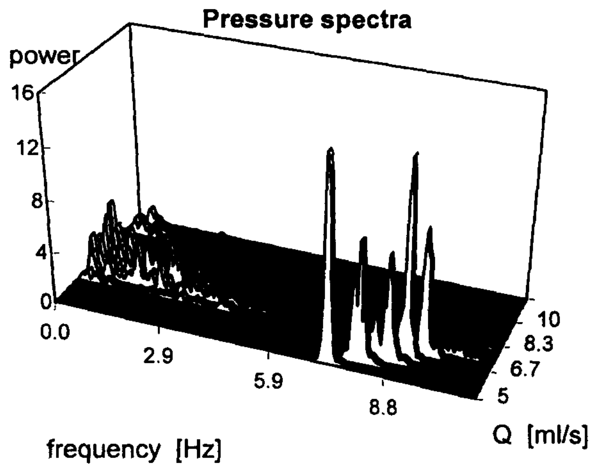


Figure 8. Comparison of experimental Kolmogorov entropy K with reconstructed value K_m based on [3] in the transition range.



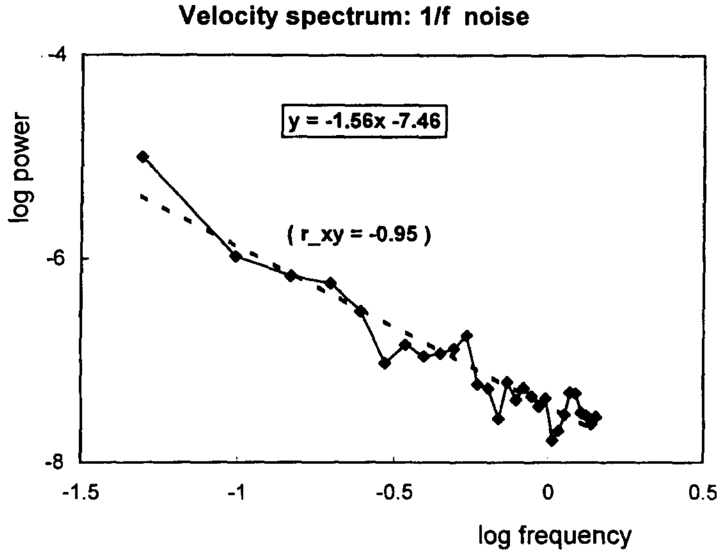


Figure 9. Typical power spectra of gas pressure signal from the chamber and liquid velocity signal from the column. Low-frequency components occur in pressure spectra in the transient region. Only low-frequency components displaying $1/f$-noise were found in velocity spectra. (Velocity spectrum corresponds to $Q = 7.49$ ml/s; $R = 0.03$ m, $H = 0.5$ m.)

4. CONCLUSIONS

The present study brings together two approaches which are often applied separately. One is the chaotic analysis of time series generated by a complex dynamic system. Mathematical chaos theory is subject on its own and chaotic signal analysis is often performed regardless of physical mechanisms producing the measurable and analysable outputs. Secondly, it is the dissection of the whole body of interconnected physical processes that contribute to the result and identification of their roles in the play. Here, an attempt has been made to make a bridge between dynamic features of liquid velocity and gas pressure signals from a model experiment and gas–liquid flow and the way how gas and liquid phases interact physically.

Experimental data showed that velocity of liquid circulations increased with the liquid column height and varied little with radial probe position. Power spectrum of velocity signal was hyperbolic in low frequency range and indicated the presence of $1/f$ noise. In the intermittent pressure signal from the gas chamber, hyperbolic distribution of periodic portion length was found, which is typical for type II and III intermittency. These facts support the conclusion that the low-frequency random liquid motion generated by bubble rise was the cause of intermittent character of the transition from bubbling to jetting regime.

Acknowledgements—This research was supported by a postdoctoral fellowship of The Royal Society, London (M. C. R.), and by the Grant Agency of the Czech Republic (grant no. 104/95/0647). The authors thank Ms V. Pěnkavová for excellent assistance in data acquisition and treatment.

REFERENCES

Baker, G. L. and Gollub, J. P. (1990) *Chaotic Dynamics: An Introduction*. Cambridge University Press, Cambridge.

Bleek, C. M. van den and Schouten, J. C. (1993a) Can deterministic chaos create order in fluidized-bed scale-up? *Chem. Eng. Sci.* **48,** 2367–2373.

Bleek, C. M. van den and Schouten, J. C. (1993b) Deterministic chaos: a new tool in fluidized bed design and operation. *Chem. Eng. J.* **53,** 75–87.

Brandstater, A. and Swinney, H. L. (1987) Strange attractors in weakly turbulent Couette–Taylor flow. *Phys. Rev. A* **35,** 2207–2220.

Castillejos, A. H. and Brimacombe, J. K. (1987) Measurement of physical characteristics of bubbles in gas–liquid plumes: Part II. Local properties of turbulent air–water plumes in vertically injected jets. *Metallurg. Trans.* **18B,** 659–671.

Deckwer, W. D. (1992) *Bubble Column Reactors*. John Wiley, Chichester.

Drahoš, J., Ruzicka, M. C., Pěnkavová, V. and Serio, C. (1996) Chaotic dynamics of bubble formation in a pool of liquid. *Proc. International Conference 'Fractals and Chaos in Chemical Engineering'*, 2–5 September 1996, Rome, Italy. World Scientific, Singapore.

Frisch, U. (1995) *Turbulence*. Cambridge University Press, Cambridge.

Haken, H. (1977) *Synergetics: An Introduction*. Springer, Berlin.

Hilborn, R. C. (1994) *Chaos and Nonlinear Dynamics*. Oxford University Press, New York.

Kastanek, F., Zahradnik, J. Kratochvil, J. and Cermak, J. (1993) *Chemical Reactors for Gas–Liquid Systems*. Ellis Horwood, Chichester.

Kennel, M. B. and Isabelle, S. (1992) Method to distinguish possible chaos from colored noise and to determine embedding parameters. *Phys. Rev. A* **46,** 3111–3118.

Kim, I., Kamotani, Y. and Ostrach, S. (1994) Modelling bubble and drop formation in flowing liquids in microgravity. *AIChE J.* **40,** 19–28.

Letzel, H. M., Schouten, J. C., Krishna, R. and Bleek, C. M. van den (1996) Characterization of regimes and regime transitions in bubble columns by chaos analysis of pressure signals. *CHISA'96 Conference*, 25–30 August 1996, Prague, Czech Republic, Summaries, Vol. 4, p. 60.

Marshall, S. H., Chudacek, M. W. and Bagster, D. F. (1993) A model for bubble formation from an orifice with liquid cross-flow. *Chem. Eng. Sci.* **48,** 2049–2059.

Miller, S. L., Miller, W. M. and McWhorter, P. J. (1993) External dynamics: a unifying physical explanation of fractals, $1/f$ noise and activated processes. *J. Appl. Phys.* **73,** 2617–2628.

Mittoni, L. J., Schwarz, M. P. and LaNauze, R. D. (1995) Deterministic chaos in the gas inlet pressure of gas–liquid bubbling systems. *Phys. Fluids* **7,** 891–893.

Mukutmoni, D. and Yang, K. T. (1995) Thermal convection in small enclosures: an atypical bifurcation sequence. *Int. J. Multiphase Flow* **38,** 113–126.

Oguz, H. N. and Prosperetti, A. (1993) Dynamics of bubble growth and detachment from a needle. *J. Fluid. Mech.* **257,** 111–145.

Rabiger, N. and Vogelpohl, A. (1983) Calculation of bubble size in the bubble and jet regimes for stagnant and flowing Newtonian liquids. *Ger. Chem. Eng.* **6,** 173–182.

Rabiger, N. and Vogelpohl, A.(1986) Bubble formation and its movement in Newtonian and non-Newtonian liquids. In *Encyclopedia of Fluid Mechanics*, Vol. 3, pp. 58–88, ed. N. P. Cheremisinoff. Gulf Publ., Houston.

Ringuet, E., Roze, C. and Gouesbet, G. (1993) In *Instabilities in Multiphase Flows*, ed. G. Gouesbet and A. Berlement, pp. 17–24. Plenum Press, New York.

Schouten, J. C. and Bleek, C. M. van den (1996) RRCHAOS: an interactive software package for deterministic chaos analysis of non-linear time series. Reactor Research Foundation, Delft University of Technology, Delft, Netherlands.

Schouten, J. C., Takens, F. and Bleek, C. M. van den (1994a) Maximum-likelihood estimation of the entropy of an attractor. *Phys. Rev. E* **49,** 126–129.

Schouten, J. C., Takens, F. and Bleek, C. M. van den (1994b) Estimation of the dimension of a noisy attractor. *Phys. Rev. E* **50,** 1851–1861.

Schuster, H. G. (1995) *Deterministic Chaos: An Introduction.* VCH Press, Weinheim, Germany.

Tritton, D. J. and Egdell, C. (1993) Chaotic bubbling. *Phys. Fluids A* **5,** 503–5.

Tsuge, H. (1986) Hydrodynamics of bubble formation from submerged orifices. In *Encyclopedia of Fluid Mechanics*, Vol. 3, pp. 191–232, ed. N. P. Cheremisinoff. Gulf Publ., Houston.

Turkoglu, H. and Farouk, B. (1996) Phase-resolved measurements in a gas-injected liquid bath. *Int. J. Heat Mass Transfer* **39,** 3401–3415.